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Brans–Dicke Cosmology with Time-Dependent Cosmological Term

Marcelo Samuel Berman[1,2]

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Berman and Som's solution for a Brans–Dicke cosmology with time-dependent cosmological term, Robertson–Walker metric, perfect fluid, and perfect gas law of state solves the horizon, homogeneity, and isotropy problems without requiring any unnatural fine tuning in the very early universe, thus being an alternative model to inflation. The model also does not need recourse to quantum cosmology, and solves the flatness and magnetic monopole problems.

1. INTRODUCTION

Bertolami (1986*a,b*) discussed a cosmological Brans–Dicke model with time-dependent cosmological term, Robertson–Walker metric, and a perfect fluid. He obtained for the time-scale factor R, the cosmological term Λ, and the scalar field ϕ the following solutions:

$$\begin{aligned} \Lambda &= Et^{-2} && \text{when} \quad p = 0 \text{ or } \rho/3 \\ R(t) &= At && \text{when} \quad p = 0 \text{ or } \rho/3 \\ \phi(t) &= St^{-1} && \text{when} \quad p = 0 \\ \phi(t) &= C't^{-2} && \text{when} \quad p = \rho/3 \end{aligned} \tag{1}$$

Here, ρ and p are rest energy density and pressure, while E, A, S, and C' are constants, and t stands for cosmic time, while

$$\phi^{-1} = \frac{3+2w}{4+2w} G$$

where G is Newton's gravitational constant and w is the coupling constant.

[1]Department of Astronomy, University of Florida, 211 SSRB, Gainesville, Florida 32611.
[2]Permanent address: Department of Exact Sciences, Faculdade de Filosofia, Ciencias e Letras da FURJ, 89200 Joinville, SC, Brazil.

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Berman and Som (1990) considered the same problem in more generality, solving it for a perfect gas law of state,

$$p = \alpha\rho \tag{2}$$

obtaining the following results:

$$\Lambda = Et^{-2} \tag{3}$$

$$R(t) = (mDt)^{1/m} \tag{4}$$

$$\phi = St^{A} \tag{5}$$

$$\rho = Ct^{-(3/m)(1+\alpha)}$$

where C, m, and D are nonnull constants, and

$$A = 2 - \frac{3(1+\alpha)}{m} \tag{6}$$

Calling the tricurvature k, we point out that for $k = \pm 1$, it was found that we must also impose $m = 1$. For $k = 0$, m may in principle assume any nonnull numerical value.

Berman and Som's solution for the scale factor is of a special kind studied earlier by Berman (1983) and Berman and Gomide (1988), to wit, constant-deceleration parameter models.

Indeed, calling $q = -\ddot{R} \cdot R/\dot{R}^2$ the deceleration parameter, we find that

$$q = m - 1 = \text{const} \tag{7}$$

2. THE SOLUTION OF THE COSMOLOGICAL "PROBLEMS"

Let us begin with the horizon problem. The horizon distance, i.e., the size of causal connected regions, is given by

$$d_H(t, t_0) = R(t) \int_{t_0}^{t} \frac{dt'}{R(t')} \tag{8}$$

For $k = \pm 1$, we have, then

$$d_H(t, t_0) = t \ln\left(\frac{t}{t_0}\right) \tag{9}$$

which diverges for $t_0 \to 0$. Thus, the model has no horizon, and we can have isotropy and large scale homogeneity.

For $k = 0$, we have: (a) If $m = +1$, the same result as above. (b) If $m \neq +1$,

$$d_H(t, t_0) = \frac{m}{m-1}(t - t_0) \tag{10}$$

If we remember that the initial singularity corresponds to $t_0 = 0$, we see that the distance horizon increases in direct proportion to the increase of the age of the universe, so that the isotropy and homogeneity of the universe as time passes tends to increase. So we would prefer $m = 1$. Let us now examine why this model, though classical, needs no quantum cosmology for the early phase.

We calculate the Planck time,

$$t_{Pl} = (G\hbar)^{1/2} = \left(\frac{a}{s}\hbar\right)^{1/2} t^{-A/2} \qquad \left(a = \frac{4+2w}{3+2w}\right) \tag{11}$$

We may suppose $A < 0$, so that when $t \to 0$, $t_{Pl} \to 0$ and we need to consider only the classical picture!

Now we would need to consider the flatness and the magnetic monopole problems, but this is not necessary, since we would proceed exactly as in Bertolami's paper (1986*b*). It should be remarked that this is a hot big-bang model. With $\alpha = \frac{1}{3}$ we would have from Stefan's law $\rho \propto T^4$, while $\rho \propto R^{-4}$ from Berman and Som's solution. Then

$$T \propto R^{-1} \tag{12}$$

3. CONCLUSIONS

The inflationary scenario may be substituted by the present model, provided that Brans–Dicke theory proves to be better than general relativity, with regard to solar system observations. The horizon, homogeneity, isotropy, and other problems were shown to be solved by this model, which is more general than Bertolami's (1986*a*,*b*). This model explains why the cosmological term is negligible today in comparison with its value in the early universe.

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